Expansion History with Decaying Vacuum: A Complete Cosmological Scenario

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ABSTRACT

We propose a novel cosmological scenario with the space-time emerging from a pure initial de Sitter stage and subsequently evolving into the radiation, matter and dark energy dominated epochs, thereby avoiding the initial singularity and providing a complete description of the expansion history and a natural solution to the horizon problem. The model is based on a running vacuum energy density ρ_Λ which evolves as a power series of the Hubble rate: $\rho_\Lambda = \rho_\Lambda(H)$. The transit from the inflation into the standard radiation epoch is universal, giving a clue for a successful description of the graceful exit. The Universe is finally driven into the present slow accelerated expansion, characterized by a residual (but dynamical) vacuum energy: $\rho_\Lambda(H) \simeq c_0 + c_2\,H^2$. While the resulting late time cosmic history is very close to the concordance ΛCDM model, the new unified framework embodies a more complete past cosmic evolution than the standard cosmology.

I. INTRODUCTION

In the current view of the cosmological history it is believed that matter and space-time emerged from a singularity and evolved through four different eras: early inflation, radiation, dark matter (DM) and dark energy (DE) dominated eras. During the radiation and DM dominated stages, the expansion of the Universe slows down while in the inflationary and DE eras it speeds up. So far there is no clear cut connection between the inflationary period and the normal Friedmann expansion. Moreover the current DE phase is also a mystery.

Over the past decade, studies of the available high quality cosmological data (supernovae type Ia, CMB, galaxy clustering, etc.) have converged towards a cosmic expansion history that involves a spatially flat geometry and a recent accelerating period of the Universe [1, 2]. This faster expansion phase has been attributed to the DE component with negative pressure. The simplest type of DE corresponds to the cosmological constant (CC) [3]. The so-called concordance model (or ΛCDM model [4]), which contains cold DM to explain clustering, flat spatial geometry and a CC, Λ, fits accurately the current observational data and thus it is an excellent candidate to be the model that describes the observed Universe. However, the ΛCDM suffers from, among others, two fundamental problems: (a) The "old" cosmological constant problem (or *fine tuning problem*) i.e., the fact that the observed value of the vacuum energy density ($\rho_\Lambda = \Lambda_0/8\pi G \simeq 10^{-47}\,GeV^4$) is many orders of magnitude below the value suggested in quantum field theory (QFT) [3], and (b) the coincidence problem [5] i.e., the fact that the (decreasing) matter energy density and the (constant) vacuum energy density happen to be of the same order just prior to the present epoch.

In this Letter we attempt to overcome, or at least to alleviate, such theoretical problems through an alternative cosmic scenario based on a time-dependent vacuum energy density, i.e. $\Lambda = \Lambda(t)$. There are a number of interesting Λ-variable models in the old [6–9] and more recent [10–12] literature. Although the functional form of $\Lambda(t)$ in most of them has usually been proposed on phenomenological grounds, as it occurs with the vast majority of DE models [13], a more fundamental approach would be desirable within QFT in curved space-time [14].

In what follows we investigate the cosmic expansion within a class of time evolving vacuum models along these lines [12], and at the same time involving ingredients capable of yielding

a smooth transition from an early de Sitter stage to a proper radiation ($a \propto t^{1/2}$) and matter ($a \propto t^{2/3}$) epochs [8].

II. THE UNIFIED DECAYING VACUUM MODEL

Consider the class of time evolving vacuum models following a power series of the Hubble rate:

$$\Lambda(H) = c_0 + c_1 H + c_2 H^2 + c_3 H^3 + c_4 H^4 + ... \tag{1}$$

with $\rho_\Lambda(H) = \Lambda(H)/(8\pi G)$ the corresponding vacuum energy density. The constant c_0 in (1) represents the dominant term at low energies (i.e. when H is near the current value H_0). The $H^k (k \geqslant 1)$ powers represent small corrections to the dominant term and provide a time evolving behavior to the vacuum energy density. The present value of the CC is not just equal to c_0, as we still have to add to it the contribution from the leading correction terms that we consider in the expansion (1). Clearly the H^2 power, with the dimensionless coefficient c_2, is singled out by the fact that it represents the maximum power of H that can contribute significantly at low energies. The dimensionful coefficient c_1 should be of order $\mathcal{O}(H_0)$ in the current Universe and hence it adds nothing essential to the behavior of $c_2 H^2$ at low redshifts [15, 16]. The minimal model of the type (1) at low energy therefore reads [12]

$$\Lambda(H) = c_0 + c_2 H^2 = \Lambda_0 + 3\,\nu\,(H^2 - H_0^2)\,, \tag{2}$$

where $\Lambda_0 = c_0 + 3\nu\, H_0^2$ is the current value of the CC and $c_2 \equiv 3\nu$. The dimensionless parameter ν remains to be determined by confronting the model with observations. Using a joint likelihood analysis of the recent supernovae type Ia data, the CMB shift parameter, and the Baryonic Acoustic Oscillations one finds that the best fit parameters for a flat universe are [15]: $\Omega_m^0 \simeq 0.27 - 0.28$ and $|\nu| = \mathcal{O}(10^{-3})$. It is remarkable that the fitted value of ν is within the theoretical expectations when this parameter plays the role of β-function of the running CC [10]. In specific frameworks one typically finds $\nu = 10^{-5} - 10^{-3}$ [17]. Intriguingly, the order of magnitude of ν actually strengthens a possible connection of these dynamical vacuum models with a potential variation of the so-called fundamental constants of Nature [18].

The novelty in the present approach is that we extend the domain of applicability of these models, namely we encompass in a single unified framework both the inflationary [8] and the

current dark energy epochs [12, 15] and essentially bridge them with the standard Friedmann regime. While the higher order powers H^k $(k > 2)$ in (1) are completely negligible at present, they can acquire a great relevance in the early universe [8]. In a minimal realization we expect that only two powers should matter: one (H or H^2) for the low energy limit or current universe, and another (H^3 or H^4) for the high energy behavior in the early universe. The linear term in H, however, is not only phenomenologically irrelevant or unfavored (depending on its use) [15, 16] but there are actually sound theoretical reasons to neglect it, as only the even powers of H can appear in the fundamental effective action of QFT in curved space-time [10, 12, 17]. Following this paradigm, we will take only the next-to-leading even power of H in the series (1), i.e. H^4, to describe the high energy regime near the inflationary epoch. This leads to the following minimal model

$$\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha\frac{H^4}{H_I^2}\,, \tag{3}$$

for a unified description of the complete cosmological history. Here α is a dimensionless constant and the scale H_I can be interpreted as the inflationary expansion rate (see below).

The Einstein equations for a spatially flat universe with a matter fluid (p_m, ρ_m) and non-vanishing CC read:

$$8\pi G\rho_{\rm tot} \equiv 8\pi G\rho_m + \Lambda \;=\; 3H^2\,, \tag{4}$$

$$8\pi G p_{\rm tot} \equiv 8\pi G p_m - \Lambda \;=\; -2\dot{H} - 3H^2\,, \tag{5}$$

from which we obtain the time evolution equation of the Hubble rate $(H \equiv \dot{a}/a)$, where the overdot denotes derivative with respect to cosmic time t. Upon using (3) and the equation of state for the non-vacuum fluid $p_m/\rho_m = \omega_m$, it follows that:

$$\dot{H} + \frac{3}{2}(1+\omega_m)H^2\left[1 - \nu - \frac{c_0}{3H^2} - \alpha\frac{H^2}{H_I^2}\right] = 0\,. \tag{6}$$

Remarkably there is the constant value solution $H^2 = (1-\nu)H_I^2/\alpha$ of this equation for the very early universe (where we can safely neglect $c_0/H^2 \ll 1$). It signals the presence of an inflationary epoch. For late times we have $H \ll H_I$, implying that the $\alpha H^4/H_I^2$ term in (3) becomes negligible and the model boils down to the low energy form (2).

We shall present below the various phases of the decaying vacuum cosmology (3), starting from an unstable inflationary phase $[a(t) \propto e^{H_I t}]$ powered by the huge value H_I presumably connected to the scale of a Grand Unified Theory (GUT) or even the Planck scale M_P, then

it deflates (with a massive production of relativistic particles), and subsequently evolves into the standard radiation ($a(t) \propto t^{1/2}$) and matter ($a(t) \propto t^{2/3}$) dominated eras. Finally, it effectively appears today as a slowly dynamical DE driven by Eq. (2).

The Hubble function and scale factor of this model in the early universe (when c_0 can be neglected) follows from direct integration of Eq.(6):

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}\,, \tag{7}$$

$$\int_{a_*}^{a} \frac{d\tilde{a}}{\tilde{a}} \sqrt{D\, \tilde{a}^{3(1-\nu)(1+\omega_m)} + 1} = \sqrt{\frac{1-\nu}{\alpha}}\, H_I\, \Delta t\,, \tag{8}$$

where $D > 0$ is a constant. Notice that a_* is the scale factor at the transition time (t_*) when the inflationary period ceases, and $\Delta t = t - t_*$ is the cosmic time elapsed since then. Using (7) and the Einstein equations (4)-(5) we may also obtain the corresponding energy densities:

$$\begin{aligned} \rho_m(a) &= \rho_I \, \frac{(1-\nu)^2}{\alpha} \frac{D a^{3(1-\nu)(1+\omega_m)}}{\left[D a^{3(1-\nu)(1+\omega_m)} + 1\right]^2} \\ \rho_\Lambda(a) = \frac{\Lambda(a)}{8\pi G} &= \rho_I \, \frac{1-\nu}{\alpha} \, \frac{\nu D a^{3(1-\nu)(1+\omega_m)} + 1}{\left[D a^{3(1-\nu)(1+\omega_m)} + 1\right]^2}\,, \end{aligned} \tag{9}$$

where $\rho_I = 3H_I^2/8\pi G$ is the primeval critical energy density associated to the initial de Sitter stage. Obviously, if $Da^{3(1-\nu)(1+\omega_m)} \ll 1$ (i.e. $t \ll t_*$) then Eq. (7) boils down to the particular solution mentioned above, in which $H = \sqrt{(1-\nu)/\alpha}\, H_I$ is constant, and $\rho_m \simeq 0$, $\rho_\Lambda \propto \rho_I$ (i.e. no matter and huge vacuum energy density). From (8) it is obvious that $a(t) \sim e^{\sqrt{(1-\nu)/\alpha}\, H_I\, \Delta t}$ and the universe then inflates. For $Da^{3(1-\nu)(1+\omega_m)} \gg 1$ (i.e. $t \gg t_*$), instead, Eqs. (9) tell us that both $\rho_m(a)$ and $\rho_\Lambda(a)$ decay as $\sim a^{-3(1-\nu)(1+\omega_m)}$ while at the same time the ratio $|\rho_\Lambda(a)/\rho_m(a)|$ remains very small, viz. of order $|\nu| \leq \mathcal{O}(10^{-3})$ [15], which insures that primordial nucleosynthesis will not be harmed at all. Furthermore, since the vacuum presumably decayed mostly into relativistic particles ($\omega_m = 1/3$) we find from Eq. (8) that in the post-inflationary regime $a \sim t^{\frac{2}{3(1-\nu)(1+\omega_m)}} \sim t^{1/2}$, i.e. we reach essentially the standard radiation epoch – confirmed, in addition, by the fact that the matter (and vacuum) energy densities $\rho_m(a)$ and $\rho_\Lambda(a)$ decay as $\sim a^{-4}$ in this period. The universe thus evolves continuously from inflation towards a standard (FLRW) radiation dominated stage, as shown in the inner plot of Fig. 1. In between these two eras, we see from the first Eq. (9) that we can have either huge relativistic particle production $\rho_r \propto a^4$ in the deflation period (namely around $Da^4 \gtrsim 1$) or standard dilution $\rho_r \propto a^{-4}$ well in the radiation era ($Da^4 \gg 1$).

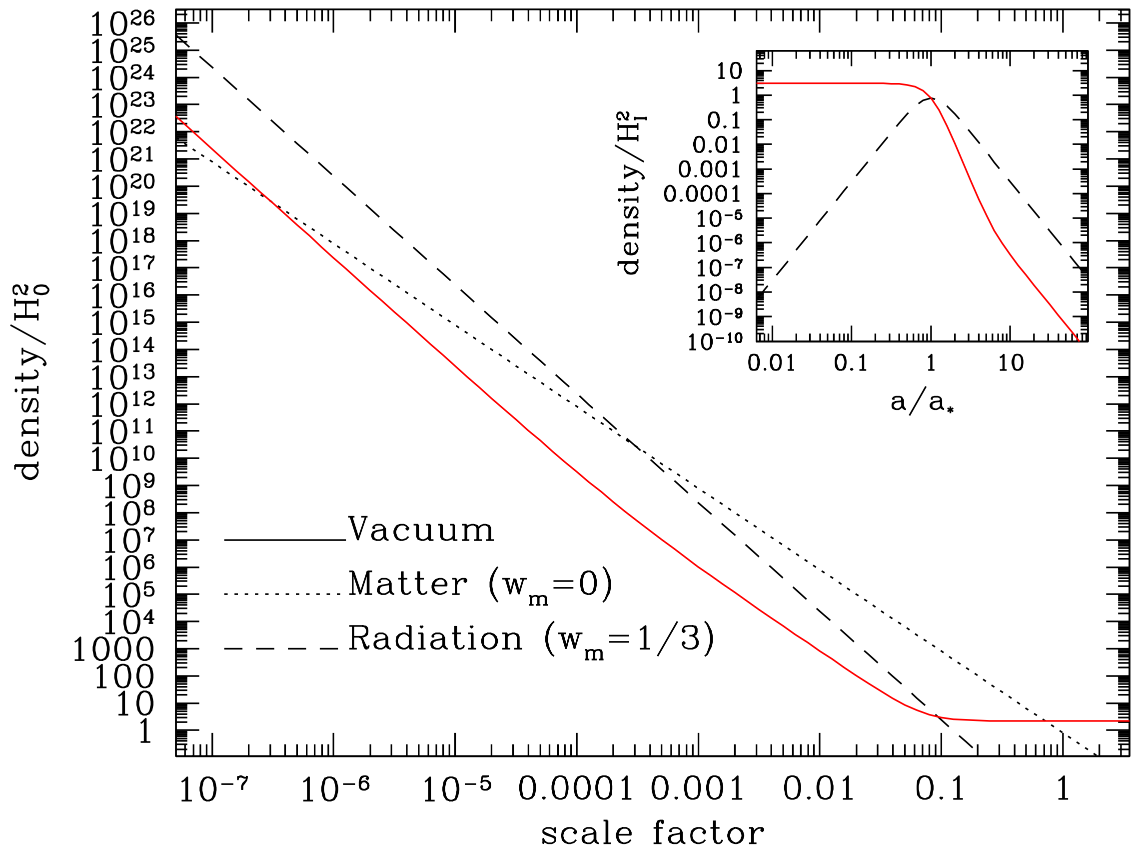

FIG. 1: Outer plot: The evolution of the radiation, matter and vacuum energy densities, for the unified vacuum model (3) in units of H_0^2. The curves shown are: radiation (dashed line), matter (dotted line) and vacuum (solid line, in red). To produce the lines we used $\nu = 10^{-3}$, $\Omega_m^0 = 0.27$, $\Omega_R^0 = (1 + 0.227N_v)\,\Omega_\gamma^0$, $\Omega_\Lambda^0 = 1 - \Omega_m^0 - \Omega_R^0$, $(N_v, \Omega_\gamma^0, h) \simeq (3.04, 2.47 \times 10^{-5}h^{-2}, 0.71)$ [2], and set $\alpha = 1$ and $D = 1/a_*^{3(1-\nu)(1+\omega_m)}$. Inner plot: the transition from the primeval vacuum epoch (inflationary period) into the FLRW radiation epoch. Same notation for curves as before, although the densities are now normalized with respect to H_I^2 and the scale factor with respect to a_* (see text). For convenience we used $8\pi G = 1$ units in the plots.

Naturally, due to the initial de Sitter phase, the model is free of particle horizons. A light pulse beginning at $t = -\infty$ will have traveled by the cosmic time t a physical distance $d_H(t) = a(t)\int_{-\infty}^{t} \frac{d\tilde{t}}{a(\tilde{t})}$, which diverges thereby implying the absence of particle horizons: the local interactions may homogenize the whole Universe. It should be clear that our accessible part of the universe is only for times $t > t_*$, i.e. $\Delta t > 0$. After inflation has occurred and the FLRW (radiation dominated) phase has been causally prepared, the cosmic time Δt can

just be called t and it is this one that parameterizes all cosmological equations in physical cosmology [4].

Although the motivation of the present model has a root in the general structure of the effective action of QFT in curved space-time, we cannot provide the latter at this point. However we can mimic it through a scalar field (ϕ) model for the interacting DE [19]. This can be useful for the usual phenomenological descriptions of the DE, and can be obtained from the usual correspondences $\rho_{\rm tot} \to \rho_\phi = \dot{\phi}^2/2 + V(\phi)$ and $p_{\rm tot} \to p_\phi = \dot{\phi}^2/2 - V(\phi)$ in Friedmann's Eqs. (4)-(5). We find $4\pi G\dot{\phi}^2 = -\dot{H}$ and

$$V = \frac{3H^2}{8\pi G}\left(1 + \frac{\dot{H}}{3H^2}\right) = \frac{3H^2}{8\pi G}\left(1 + \frac{1}{3}\frac{d\ln H}{d\ln a}\right) . \tag{10}$$

We can readily work out the effective potential for our model (3) in the early universe as a function of the scale factor. Neglecting small $\mathcal{O}(\nu)$ corrections, which we have seen are not important in the early stages, we arrive at

$$V(a) = \frac{\rho_I}{\alpha}\,\frac{1 + Da^4/3}{(1 + Da^4)^2}. \tag{11}$$

From this expression it becomes clear that the potential energy density remains large and constant while $a \ll D^{-1/4}$ (i.e. before the transition from inflation to the deflationary regime). Afterwards (when $a \gg D^{-1/4}$) it decreases steadily as $V(a) \sim a^{-4}$, hence as radiation. This confirms, in the effective scalar field language, the previously described decay of the vacuum energy into relativistic matter in our original framework.

III. FROM EARLY UNIVERSE TO THE PRESENT VACUUM-MATTER ERA

Let us finally discuss the cosmic evolution after the inflationary period is left well behind, i.e. when $H \ll H_I$. In this case we recover the behavior of the original running vacuum model [10, 12]. The cosmic fluid will be first in the radiation dominated epoch ($\omega_m = 1/3$) and later on in the cold matter dominated epoch ($\omega_m = 0$). The evolution equation for the Hubble function (6) in each one of these epochs reads:

$$\dot{H} + \frac{3}{2}(1 + \omega_m)(1 - \nu)H^2 - \frac{1 + \omega_m}{2}\,c_0 = 0\,. \tag{12}$$

Note that during the radiation-vacuum phase ($\omega_m = 1/3$) we can neglect the c_0 contribution in the above equation so that the evolution of the scale factor is given by the expression earlier

deduced starting from the de Sitter phase ($a \sim t^{1/2}$). However, this is not possible when we are deeper in the vacuum-cold matter dominated period and specially near the present time, as its value is close to the measured CC up to a small $\mathcal{O}(\nu)$ correction: $c_0 = \Lambda_0 - 3\nu\, H_0^2$. Trading the cosmic time for the scale factor and using the redshift variable $1+z = 1/a$ with the boundary condition $H(z=0) = H_0$, one finds the solution of Eq. (12) for the late stages ($\omega_m = 0$):

$$H^2(z) = \frac{H_0^2}{1-\nu}\left[(1-\Omega_\Lambda^0)(1+z)^{3(1-\nu)} + \Omega_\Lambda^0 - \nu\right] \,, \tag{13}$$

where $\Omega_\Lambda^0 = \Lambda_0/3H_0^2 = 8\pi\, G\, \rho_\Lambda^0/3H_0^2$. In a similar way we can obtain the matter and vacuum energy densities as a function of the redshift:

$$\rho_m(z) \; = \; \rho_m^0 (1+z)^{3(1-\nu)} \,, \tag{14}$$

$$\rho_\Lambda(z) \; = \; \rho_\Lambda^0 + \frac{\nu\, \rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right] . \tag{15}$$

From these equations it is clear that for $\nu = 0$ we recover exactly the ΛCDM expansion regime, the standard scaling law for nonrelativistic matter and a strictly constant vacuum energy density $\rho_\Lambda = \rho_\Lambda^0$ (hence $\Lambda = \Lambda_0$). Recalling that $|\nu|$ is found to be rather small when the model is confronted to the cosmological data, $|\nu| \leq \mathcal{O}(10^{-3})$ [15], it is obvious that at the present time this vacuum model is almost indistinguishable from the concordance ΛCDM model, except for its mild dynamical behavior which leads to an effective equation of state for the vacuum energy that can mimic quintessence or phantom energy [12]. Finally, at very late time we get an effective cosmological constant dominated era, $H \approx H_0 \sqrt{(\Omega_\Lambda - \nu)/(1-\nu)}$, that implies a pure de Sitter phase of the scale factor.

In Figure 1 we display, in addition to the mentioned details of the early stages of the cosmic evolution (inner plot), also the numerical analysis describing the transition of the energy densities from the radiation epoch into the matter dominated period, leading finally to the asymptotic de Sitter phase beyond our time (outer plot). The equality time of matter and radiation, $\rho_m(z_{\rm eq}) = \rho_R(z_{\rm eq})$ [2], is also marked in the plot. It corresponds to $z_{\rm eq} \simeq 3200$, thus with no essential change with respect to the ΛCDM. On the other hand for $z \lesssim 10$ (or $a \gtrsim 0.1$) the vacuum energy density appears as effectively frozen to its nominal value, $\rho_\Lambda(z) \simeq \rho_\Lambda^0$, but still displaying a slow cosmic evolution as in (15). The considered time varying vacuum model therefore provides a description of the fact that the matter density and the DE density are of the same order prior to the present epoch, as well as of the onset of the late time cosmic acceleration. From the viewpoint of structure formation, in order

to have growth of spatial density fluctuations the DM part should be capable of clustering and providing the formation of galaxies, while the vacuum density should be low enough to allow cosmic structures to form. Obviously, this situation is guaranteed in our model during the matter dominated era.

We may wonder if the virtues of the cosmological picture under consideration are confined to the peculiarities of the specific model (3). Remarkably, the latter is just the minimal or "canonical" implementation of the general class of models (1) based on the even powers of H which are favored from the point of view of the general covariance. It turns out that many other models of this class can still do the job. Interestingly, as it will be shown elsewhere [20], all the vacuum models of the form

$$\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha \frac{H^n}{H_I^{n-2}} \qquad (n > 2) \tag{16}$$

(of which the model under study is just the particular case $n = 4$) perform automatically the transition from an unstable inflationary phase (deflation) into the standard FLRW radiation dominated era, irrespective of the differences in other details [21]. The generalized expression above ensues that that there is a large class of dynamical vacuum models which could be instrumental to effectively implement the long sought-for mechanism of "graceful exit" plaguing many inflationary cosmologies [4].

IV. CONCLUSIONS

To summarize, in this work we have put forward a new global cosmological scenario which provides a consistent and rather complete account of the expansion history of the Universe. Although it can be proposed on mere phenomenological grounds, it is interesting to note that a dynamical vacuum framework based on the H^2 and H^4 terms is compatible with the general form of the effective action of QFT in curved space-time. There is actually a large class of generalized models of this sort, Eq. (16), all of them sharing a similar phenomenological behavior [20], to wit: 1) the Universe starts from an inflationary non-singular state, thus overcoming the horizon problem; 2) the early inflationary regime has a natural (universal) ending into the radiation phase; and 3) the small current value of the vacuum energy density ($\rho_\Lambda \simeq 10^{-47}\, GeV^4$) can be conceived as a result of the massive disintegration of the vacuum into matter in the primordial stages, leaving a small ("fossil" [17]) and very smooth dynamical

component $\rho_\Lambda = c_0 + c_2\, H^2$ at present. The upshot is a new unified vacuum picture of the cosmic evolution spanning from the early inflation period to the late dark energy era and deviating only very mildly from the observed ΛCDM behavior. The amplitude of the expected deviations ($|\nu| \sim 10^{-3}$) can decide which is the more realistic description of the vacuum at late stages, and, potentially, it may provide an indication favoring the complete decaying vacuum scenario proposed here. Finally, we also emphasize that the model makes definite predictions and is also in agreement with the basic cosmological probes [20].

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